FORM 3KBP

The College Board

Advanced Placement Examination

CALCULUS AB

SECTION II

This green insert may be used for reference and/or scratchwork as you answer the free-response questions, but be sure to show all your work and your answers in the pink booklet. No credit will be given for work shown on this green insert.

Princeton, N.J. 08541

Section II consists of six free-response questions. Following are the 1995 directions for Section II. A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS OR PARTS OF QUESTIONS ON THIS SECTION OF THE EXAMINATION.

SHOW ALL YOUR WORK. Indicate clearly the methods you use because you will be graded on the correctness of your methods as well as on the accuracy of your final answers. If you choose to use decimal approximations, your answer should be correct to three places after the decimal point.

Notes:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) Unless otherwise specified, answers can be given in unsimplified form. For example, if $\frac{7}{3}$ is the correct answer, then $1+\frac{8}{6}$ would be acceptable. The three decimal place approximation 2.333 would also be acceptable. Furthermore, algebraic answers need not be simplified. For example, $2x + 3x$ is as acceptable as $5x$.

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CALCULUS AB

SECTION II

Time—1 hour and 30 minutes

Number of problems—6

Percent of total grade—50

SHOW ALL YOUR WORK. INDICATE CLEARLY THE METHODS YOU USE BECAUSE YOU WILL BE GRADED ON THE CORRECTNESS OF YOUR METHODS AS WELL AS ON THE ACCURACY OF YOUR FINAL ANSWERS.

Notes: (1) In this examination $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e). (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. Let f be the function given by $f(x) = \sqrt{x^4 - 16x^2}$.

   (a) Find the domain of f.

   (b) Describe the symmetry, if any, of the graph of f.

   (c) Find $f'(x)$.

   (d) Find the slope of the line normal to the graph of f at $x = 5$.

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2. A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 1 - \sin(2\pi t)$.

   (a) Find the acceleration $a(t)$ of the particle at any time t.

   (b) Find all values of t, $0 \leq t \leq 2$, for which the particle is at rest.

   (c) Find the position $x(t)$ of the particle at any time t if $x(0) = 0$.

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3. Let $f(x) = \cos x$ and $g(x) = x^2 - 1$.

   (a) Find the coordinates of any points of intersection of f and g.

   (b) Find the area bounded by f and g.

   (c) Find the volume generated when the region in part (b) is rotated around the y-axis.

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4. Let f be the function defined by $f(x) = 2xe^{-x}$ for all real numbers x.

   (a) Write an equation of the horizontal asymptote for the graph of f.

   (b) Find the x-coordinate of each critical point of f. For each such x, determine whether $f(x)$ relative maximum, a relative minimum, or neither.

   (c) For what values of x is the graph of f concave down?

   (d) Using the results found in parts (a), (b), and (c), sketch the graph of $y = f(x)$ in the xy-provided below. Note: The xy-plane is provided in the pink test booklet only.

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5. Let R be the region in the first quadrant under the graph of $y = \dfrac{x}{x^2 + 2}$ for $0 \leqq x \leqq$

(a) Find the area of R.

(b) If the line $x = k$ divides R into two regions of equal area, what is the value of k?

(c) What is the average value of $y = \dfrac{x}{x^2 + 2}$ on the interval $0 \leqq x \leqq \sqrt{6}$?

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6. Let f be a differentiable function, defined for all real numbers x, with the following properties.

(i) $f'(x) = ax^2 + bx$

(ii) $f'(1) = 6$ and $f''(1) = 18$

(iii) $\int_1^2 f(x)dx = 18$

Find $f(x)$. Show your work.

END OF EXAMINATION.